**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS:** B

(1-pnorm(50,45,8)) = 0.265985529048701

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**ANS:** False

Around 70% of the data falls within one standard deviation of the mean (µ+= 38+6=44)

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS**: True

Z=(X-µ)/ *σ*

P(X≤30)=p(Z≤(30-38)/6)=p(Z≤-1.33)= 0.0918(using z table)

Expected count=0.0918\*400= 36.72

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS**: 2 *X*1  will be greater scale version than *X*1 + *X*2 . If *X*1 and *X*2 are normally distributed then the sum of the random sample will be exactly same

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS:** D

qnorm(0.995,100,20)

qnorm(0.005,100,20)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANS:**

To find a Rupee range (centered on the mean) that contains 95% probability for the annual profit of the company, we can use the Empirical Rule (also known as the 68-95-99.7 rule). According to this rule, for a normal distribution:

* Approximately 68% of the data falls within one standard deviation from the mean.
* Approximately 95% of the data falls within two standard deviations from the mean.
* Approximately 99.7% of the data falls within three standard deviations from the mean.

Given that Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42), we can calculate the total profit's mean and standard deviation:

1) Mean of the total profit:

**Mean**total = **Mean**Profit1 + **Mean**Profit2

**Mean** total=5+7 = 22

2) Variance of the total profit:

**Var**total **= Var**Profit1 **+ Var**Profit2

**Var**total **=** 32 + 42 = 74

3) Standard deviation of the total profit:

**SD**total = **√' Var**total = **√'**74

Now, we can use the Empirical Rule to find the Rupee range:

95% probability falls within two standard deviations from the mean. Therefore, the range would be **Mean**total ± 2 × **SD**total

​After calculating this range, you can convert it to Rupees using the given conversion rate ($1 = Rs. 45).

1. Specify the 5th percentile of profit (in Rupees) for the company

**ANS:**To find the 5th percentile of the profit for the company, you can use the quantile function. The formula for the total profit is:

Total Profit = Profit1 + Profit2

Given that Profit1 ∼ N(5,32) Profit1∼N(5,32) and Profit2∼N(7,42), the mean and standard deviation of the total profit (Total Profit) are calculated as follows:

1)Mean of the total profit:

**Mean**total **= Mean**Profit1 + **Mean**Profit2

**Mean**total= 5 + 7 =12

2)Standard deviation of the total profit:

**SD**total = √'**Var**Profit1 + **Var**Profit2

**SD**total = √' 32 + 42 = 74

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1. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS:** To determine which division has a larger probability of making a loss, we can look at the mean and standard deviation of each division's profit distributions.

Profit1:

* Mean (μProfit1) = 5
* Standard Deviation (σProfit1 ) = √'32

​ Profit2:

* Mean (μProfit2) = 7
* Standard Deviation (σProfit2 ) = √'42

A division is likely to make a loss if the profit is below zero. We can calculate the probability of making a loss for each division by finding the Z-score for zero profit (Z = Profit−Mean/Standard Deviation) and then using a standard normal distribution table or a statistical function.

Let's calculate the probabilities:

1) Probability of profit1 making a loss:

P( Profit1 < 0 ) = P ( Z < 0-5/ √'32 )

2) Probability of profit2 making a loss:

P( Profit2 < 0 ) = P ( Z < 0-7/ √'42 )

Compare the two probabilities. The division with the higher probability of making a loss has a greater chance of facing losses in a given year.